Michael Rulle, *MSR Investments, LLC*, June 2014

Sharpe Ratios Reported by Hedge Fund Indices Underestimate Annual Standard Deviation

> ***'Letting S_1 and S_T denote the Sharpe Ratios for 1 and T periods, respectively, it follows that: S_T =SQRT(T)*S_1. In practice, the situation is likely to be more complex…underlying differential returns may be serially correlated. Even if the underlying process does not involve serial correlation, a specific ex post sample may.'***
>
> From "The Sharpe Ratio" by William Sharpe in the Fall 1994 issue of *The Journal of Portfolio Management*, page 49.

Sharpe Ratios reported by hedge fund indices are commonly calculated without regard to their monthly serial correlation. This practice often has the effect of understating a return stream's annual standard deviation and thus overestimating a particular strategy's risk-adjusted return. The "square root of time" formula as an estimator for the annual standard deviation of hedge fund returns is imprecise when serial correlation exists in the data. Using the proper formula to calculate a hedge fund strategy's risk adjusted return is crucial in determining whether or not a particular hedge fund strategy is providing the benefits it purports to.

We are not claiming a new discovery. We are simply restating what has been known and written about by the financial academic community for at least the last 15 years. Countless articles shed light on the subject - a list of which is provided at the end of this essay. Sharpe Ratios reported by almost all hedge fund indices (and by deduction, hedge fund managers themselves) systematically understate risk when positive serial correlation is present but not accounted for.

To demonstrate this point, we analyzed the Credit Suisse Broad Hedge Fund Index, the largest capital weighted index by asset size among all hedge fund indices with monthly return data extending back to 1994 and a publicized lifetime Sharpe Ratio of 0.80 through March 2014.[1]

How is the Sharpe Ratio for the Credit Suisse Broad Hedge Fund Index currently calculated? Credit Suisse divides the index's annual geometric mean return less the Risk Free Rate (Credit Suisse uses the annualized rolling 90 day T-bill rate as an estimate for the Risk Free Rate) by the index's annualized monthly standard deviation:

$$= \frac{\text{(Annual Geometric Mean Return - Annualized rolling 90 day T-bill Rate)}}{\text{Standard Deviation of Monthly Returns * SQRT (12)}}$$

$$= \frac{(8.65\% - 2.82\%)}{2.10\% * \text{SQRT } (12)}$$

$$= \frac{5.82\%}{7.28\%}$$

$$= 0.80$$

[1] Credit Suisse Hedge Fund Indexes, http://www.hedgeindex.com (accessed April 21, 2014).

Notice the numerator and the denominator in the above equation. We are comparing the numerator - a calculated annual geometric mean return - to the denominator - an "annualized" monthly standard deviation.

The distinction is important, particularly because you can easily calculate the standard deviation of the index's 20 years of annual returns (1994-2013) and discover that the actual measured standard deviation of annual returns is significantly higher (10.92%) than the annualized monthly standard deviation calculated above by Credit Suisse (7.28%).

Taking it one step further, if you were to insert the standard deviation of annual returns (10.92%) into the equation above in place of the annualized monthly standard deviation (7.28%), you would calculate a Sharpe Ratio of 0.53 as demonstrated below:

$$= \frac{\text{(Annual Geometric Mean Return - Annualized rolling 90 day T-bill Rate)}}{\text{Standard Deviation of Annual Returns}}$$

$$= \frac{(8.65\% - 2.82\%)}{10.92\%}$$

$$= \frac{5.82\%}{10.92\%}$$

$$= 0.53$$

Using the annualized monthly standard deviation, as opposed to the annual standard deviation, results in a Sharpe Ratio that is ***overstated by 50%***. Using an inflated Sharpe Ratio may not only result in a material understatement of potential loss but can also lead investors to value highest those strategies with the highest positive serial correlation and, therefore, the most hidden risk. *In a future essay, we intend to demonstrate in more detail the negative impact created by using the "*square root of time" *formula on portfolio construction when positive serial correlation is ignored.*

How did the investment community come to make this error?

Over the years, the financial community has become so accustomed to the term "annualized standard deviation" that it has improperly come to equate it with "annual standard deviation". The reason we use "annualized standard deviation" is to ***estimate*** the annual standard deviation.

We will describe how and why this phenomenon has arisen. Regardless, in the event that monthly data does not exhibit serial correlation, a calculated annual standard deviation is just as suitable an estimator of the ***population*** annual standard deviation as a formula that uses an annualized monthly standard deviation. In the event that serial correlation is present in the monthly data, it is preferable to use the annual data to calculate an annual standard deviation. It does not matter how many years of data are available (as long as it is 2 or greater) because "24 months" (or 36, 60, 240, etc.) of data results in a comparable annual standard error as 2 years (or 3, 5, 10, etc.). That being said, it is still beneficial to look at both annual standard deviations and annualized monthly standard deviations properly adjusted for serial correlation when calculating Sharpe ratios.

Perhaps one of the reasons why the practice of using annualized monthly standard deviations to estimate population annual standard deviations has continued is that the math behind it, while not difficult, is also not intuitive. Interestingly, it was Albert Einstein who discovered the "square root of time" concept. In 1905, Einstein realized that Brownian particles dispersed in a random walk in direct proportion to the square root of time. Notably, Einstein also recognized that a necessary condition to the expansion of standard deviation as a function of the square root of time was that the variable to which it was being applied needed to be independent and identically distributed (i.i.d.).

Quantitative practitioners of finance borrow heavily from physics. The academic financial revolution of the 1950s-1970s led to the measurement of how stock market returns dispersed through time and an application for Einstein's formula was found. ***As long as returns followed a "random walk" (a lognormal distribution),*** one could estimate the standard deviation of returns over every time frame as long as one knew the standard deviation of returns for any time frame. For example, if the one trading day standard deviation of returns is 1%, and the daily returns follow a random walk, then the twenty trading day standard deviation is 4.47% as demonstrated below:

20 day Standard Deviation = 1 day Standard Deviation * SQRT (20)

= 1% * SQRT (20)

= 4.47%

And so it follows that the one year standard deviation of returns is 16% (256 trading days) and so on.

Applying Einstein's formula for annualized standard deviation to monthly return numbers enables Credit Suisse to generate an annualized monthly standard deviation for the Credit Suisse Broad Hedge Fund Index of 7.28% instead of the measured annual standard deviation of 10.92%. Importantly, however, the square root of time discovery by Einstein only applies to measured data which follows a strict random walk. If a return series is not log-normally distributed, then Einstein's formula is not applicable.

The simple fact is that ***most hedge fund strategy returns do not follow a random walk*** (Dedicated Short Bias, Equity Market Neutral and Managed Futures being the exceptions). The serial correlations of most hedge fund returns are too large and stable for this to be an accident or a function of randomness. In other words, it is likely that serial correlation, far from being an accidental artifact, is rather a direct outcome of the hedge fund styles themselves. Andrew Lo and many other leading authorities on hedge funds believe that serial correlation is a function of illiquid securities and "fair value" marking. Regardless of the reasons, the returns most hedge funds produce are subject to serial correlation as detailed in Table 1 below:

Table 1: Serial Correlation of Hedge Fund Strategies

INDEX SUBSECTORS*	Credit Suisse Broad Hedge Fund Index Jan 1994 - Mar 2014	Credit Suisse AllHedge Index Oct 2004 - Mar 2014	Credit Suisse Broad Hedge Fund Index Oct 2004 - Mar 2014
Convertible Arbitrage	0.55	0.59	0.55
Dedicated Short Bias	0.09	0.03	0.08
Emerging Markets	0.30	0.30	0.29
Equity Market Neutral	0.07	0.06	0.04
Event Driven	0.36	0.31	0.36
Event Driven Distressed	0.39	*N/A*	0.52
Event Driven Multi-Strategy	0.31	*N/A*	0.27
Event Driven Risk Arbitrage	0.28	*N/A*	0.26
Fixed Income Arbitrage	0.52	0.58	0.57
Global Macro	0.09	0.31	0.20
Long/Short Equity	0.19	0.26	0.23
Managed Futures	0.03	0.03	-0.02
Multi-Strategy	0.32	0.56	0.52
INDEX OVERALL	**0.20**	**0.45**	**0.37**

**Serial correlations were calculated for the Credit Suisse Broad Hedge Fund Index and the Credit Suisse AllHedge Index and their corresponding index subsectors using monthly return data provided on the Credit Suisse website (www.hedgeindex.com) and a one month lag period.*

Recognizing that the vast majority of hedge fund returns do display significant positive serial correlation, it is important to account for the non-random walk nature of their returns in the estimation of annual volatility so as not to underestimate annual volatility.

Statistician Gerald van Belle developed a simple formula (which can be used for any regression with serial correlation when used to calculate a t-statistic) which we have adapted for annualizing monthly standard deviations that accounts for serial correlation, when serial correlation is present[2]. The steps are as follows:

1. Calculate the serial correlation ("v") of the monthly return stream using a one month lag period;

2. Calculate the monthly standard deviation of the return stream;

3. Multiply the monthly standard deviation of the return stream by the square root of time (SQRT (12)); and

4. Divide the result calculated in Step 3 by SQRT((1-v)/(1+v)) where "v" is the measured serial correlation calculated in Step 1.

[2] Gerald van Belle. *Statistical Rules of Thumb* (Wiley Series in Probability and Statistics). Wiley-Interscience, 1st edition, March 2002, http://www.vanbelle.org (accessed April 30, 2014).

Note that if serial correlation is zero, Step 4 results in the same number generated in Step 3. Using the van Belle adjustment for volatility, we re-calculated the Sharpe Ratios for the Credit Suisse Broad Hedge Fund Index and its corresponding index subsectors. Results are displayed in Table 2 below:

Table 2: Sharpe Ratios of Hedge Fund Strategies using three different calculation techniques:

A. Using annualized monthly standard deviation

$$= \frac{\text{(Annual Geometric Mean Return - Annualized rolling 90 day T-bill Rate)}}{\text{Standard Deviation of Monthly Returns * SQRT (12)}}$$

B. Using the van Belle adjusted annualized monthly standard deviation

$$= \frac{\text{(Annual Geometric Mean Return - Annualized rolling 90 day T-bill Rate)}}{\text{(Standard Deviation of Monthly Returns * SQRT (12)) / SQRT((1-v)/(1+v))}}$$

C. Using annual standard deviation

$$= \frac{\text{(Annual Geometric Mean Return - Annualized rolling 90 day T-bill Rate)}}{\text{Standard Deviation of Annual Returns}}$$

	Credit Suisse Broad Hedge Fund Index					
	1994 - 2013			2004 - 2013		
INDEX SUBSECTORS*	**A**	**B**	**C**	**A**	**B**	**C**
Convertible Arbitrage	0.69	0.37	0.30	0.36	0.19	0.15
Dedicated Short Bias	-0.50	-0.46	-0.53	-0.62	-0.57	-0.57
Emerging Markets	0.33	0.24	0.23	0.65	0.48	0.38
Equity Market Neutral	0.22	0.21	0.18	-0.13	-0.13	-0.12
Event Driven	1.09	0.75	0.61	0.99	0.68	0.52
Event Driven Distressed	1.21	0.79	0.64	1.10	0.61	0.52
Event Driven Multi-Strategy	0.94	0.68	0.55	0.91	0.68	0.51
Event Driven Risk Arbitrage	0.89	0.67	0.69	0.84	0.65	0.68
Fixed Income Arbitrage	0.47	0.26	0.24	0.38	0.20	0.18
Global Macro	0.91	0.83	0.77	1.26	1.03	1.08
Long/Short Equity	0.70	0.57	0.47	0.68	0.53	0.45
Managed Futures	0.20	0.19	0.26	0.15	0.14	0.20
Multi-Strategy	1.04	0.74	0.64	0.88	0.50	0.41
INDEX OVERALL	**0.80**	**0.65**	**0.54**	**0.82**	**0.55**	**0.45**

**Sharpe Ratios were calculated for the Credit Suisse Broad Hedge Fund Index and its subsectors using monthly return data provided on the Credit Suisse website (www.hedgeindex.com) and a one month lag period.*

As you can see by looking at Table 2 above, the Sharpe Ratio for the Credit Suisse Broad Hedge Fund Index and its corresponding subsectors declines significantly after accounting for serial correlation (Dedicated Short Bias, Equity Market Neutral and Managed Futures being the exceptions). Hedge fund Sharpe Ratios that do not take positive serial correlation into account understate the amount of risk involved in an investment strategy.

To summarize, the formula currently being used to calculate the Sharpe Ratios of hedge fund indices:

$$= \frac{\text{Annual Geometric Mean Return - Risk Free Rate}}{\text{Annualized Monthly Standard Deviation}}$$

assumes monthly hedge fund returns follow a random walk. As we have demonstrated in Table 1 above, however, the majority of hedge fund returns display a significant amount of positive serial correlation. By not taking positive serial correlation into account, the true volatility of annual returns is underestimated and a Sharpe Ratio is produced that implicitly understates the risks associated with "smoothed earnings". In many cases, hedge funds smooth earnings due to the illiquid nature of the securities they trade (small cap stocks, convertible bonds, long/short merger arbitrage positions, high yield credit, structured derivatives of any kind, out of the money options, mortgage securities, the list goes on). Illiquidity makes it difficult for hedge fund managers to utilize valuation methods other than "fair market" valuations. While there is nothing inherently wrong with this practice, using imprecise arithmetic can hide these risks. Using the appropriate Sharpe Ratio calculation i.e. the formula that incorporates adjustments for serial correlation, will effectively reveal the true volatility of returns.

In conclusion, we believe that the most commonly used method to calculate a hedge fund strategy's Sharpe Ratio is incomplete as it does not take serial correlation into account and thus misstates that strategy’s true investment risk. The van Belle formula (and similar formulas developed by Andrew Lo and other industry professionals) is a more accurate method by which to estimate annual standard deviation when using monthly returns (in addition to calculating annual standard deviation using annual returns). We believe it is critical for the investment community to take serial correlation into consideration when comparing the risk-adjusted returns of various investment strategies. Based on the analysis we provided in this essay, we believe that the attractiveness of many hedge fund styles is materially impacted depending on whether or not the correct Sharpe Ratio formula is applied to a particular strategy's reported return stream.

References:

De Souza, C. and Gokcan, S. "Hedge Fund Volatility: It's Not What You Think It Is," *AIMA Journal*, September 2004.

Di Cesare, A., Stork, P. and de Vries, C. "Risk Measures for Autocorrelated Hedge Fund Returns," *Banca D'Italia Eurosistema,* working paper #831 (2011).

Gallais-Hamonno, G. and Nguyen-Thi-Thanh, H. "The Necessity to Correct Hedge Fund Returns: Empirical Evidence and Correction Method," Universite Libre de Bruxelles, CEB Working Paper 07-034 (2007).

Getmansky, M., Lo. A., and Makarov, I. "An Econometric Model of Serial Correlation and Illiquidity in Hedge Fund Returns," *Journal of Financial Economics,* vol. 74 (2004): 529-609.

Lo, A. "The Statistics of Sharpe Ratios," *Financial Analysts Journal,* vol. 58, no. 4 (July/August 2002): 36-52.

Pav, S. "Notes on the Sharpe Ratio," http://cran.r-project.org/web/packages/SharpeR/vignettes/SharpeRatio.pdf.

Sharpe, William. "The Sharpe Ratio," *The Journal of Portfolio Management*, Fall 1994: 49-58.